Indian Statistical Institute, Bangalore Centre
M.Math I Year, First Semester
Semestral Examination - 2013-2014
Analysis of Several Variables

Time: 3 Hours   November 15, 2013   Instructor: T.S.S.R.K. Rao

8*8=64
Maximum Score = 60

1. Let $f : \mathbb{R}^2 \longrightarrow \mathbb{R}$ be defined by $f(x,y) = (x^2 + y^2)\ \sin \frac{1}{x^2+y^2}$, for $(x,y) \neq (0,0)$ and $f(0,0) = 0$. Show that f is differentiable at $(0,0)$ but the partial derivatives are not bounded in any neighbourhood of $(0,0)$.

2. Find and classify the extreme values (if any) of $f(x,y) = x^2 + y^2 + x + y + xy$. State all the results needed for your conclusions.

3. Show that $\{(x_1, \sin^2 x_1, x_2, -x_3) : x_i \in [0, 2\pi], 1 \leq i \leq 3\}$ is a set of content zero in $\mathbb{R}^4$.

4. Let $D = \{(x,y) \in \mathbb{R}^2\ :\ 0 \leq x \leq 1,\ 0 \leq y \leq 3x + 2\}$. Show that Area of $D = 7/2$. State the theorem you need to arrive at this conclusion.

5. Let $\Phi : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ be an affine transformation. Show that there exists a $L > 0$ such that
$|| \ \Phi(x,y,z) - \Phi(x^1, y^1, z^1)|| \leq L\ ||\ (x,y,z) - (x^1, y^1, z^1)||$
for all $(x,y,z), (x^1, y^1, z^1) \in \mathbb{R}^3$.

6. Define $T\ :\ [0,1] \times [0, 2\pi] \to \mathbb{R}^2$ by $T(\gamma, \theta) = (\gamma\ \cos\theta,\ \gamma\ \sin\theta)$. Show that the range of T is the closed unit disc D and that T is one - one in the interior of the rectangle. Let f be a continuous function on D with compact support contained in $T((0,1) \times (0, 2\pi))$. Derive the formula $\int\limits_D f(x,y)dxdy = \int\limits_0^1 \int\limits_0^{2\pi} f(T(\gamma,\theta))\gamma\ d\theta d\gamma$.

7. Define a differentiable form in $\mathbb{R}^3$, also define addition and multification of forms. Derive the distributive law $(\omega_1 + \omega_2) \wedge\ \lambda = (\omega_1 \wedge\ \lambda) + (\omega_2\ \wedge\ \lambda)$, while stating all the conditions needed for the validity of this formula.

8. Let $k \geq 2$ and $\sigma = [p_0, p_1, \cdots p_k]$ be a positively oriented affine k-simplex. Use the definition of the boundary to derive that $\partial^2\sigma = \partial(\partial\sigma) = 0$.